# Performance Based Learning and Assessment Task

# *A Day at the Beach!*

**I. ASSESSSMENT TASK OVERVIEW & PURPOSE:**

The goal of this activity is to calculate volume and complete problems involving volume and rate. Students will be responsible for using similar triangles and Pythagorean Theorem to solve for missing measurements and find volume in a real life setting using a pail and water analogy. Students will choose and justify the best object to quickly fill a pail. They will determine the amount of time to empty the pail based on the rate the pail is losing sand and will derive a formula to determine the volume of a frustum.

**II. UNIT AUTHOR:**

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**III. COURSE:**

Geometry

**IV. CONTENT STRAND:**

Geometry, Measurement

**V. OBJECTIVES:**

Students will be able to**:** 1)Find the volume of cylinders, pyramids, and cones, 2)Use similar triangles to find missing measurements of pail, 3)Use the Pythagorean Theorem to solve for height, 4)Determine which container will hold the most volume, 5)Calculate time based on volume and rate, 6)Derive the formula for volume of a frustum, 7)Create a sketch of a sand castle and calculate its volume

**VI. REFERENCE/RESOURCE MATERIALS:**

Students will use: Classroom set of Geometry SOL formula sheets, Classroom set of graphing calculators, Classroom set of worksheets

**VII. PRIMARY ASSESSMENT STRATEGIES:**

The task includes an assessment component that performs two functions: (1) for the student it will be a checklist and provide a self-assessment and (2) for the teacher it will be used as a rubric. Students will use representative shapes and formulas to simulate filling objects with sand. They will need to justify the proper strategies to correctly estimate the volume.

**VIII. EVALUATION CRITERIA:**

Students will follow a guided activity worksheet and will be graded based upon the attached rubric

**IX. INSTRUCTIONAL TIME:**

Sixty to seventy-five minutes

# A Day at the Beach!

**Strand**
Geometry, Measurement

**Mathematical Objective(s)**
Students will be able to: Find the volume of cylinders, pyramids, and cones, use similar triangles to find missing measurements of pail, use the Pythagorean Theorem to solve for height, determine which container will hold the most volume, calculate time based on volume and rate, derive the formula for volume of a frustum, and create a sketch of a sand castle and calculate its volume

**Related SOL**

- G.8 (The student will solve real-world problems involving right triangles by using the Pythagorean Theorem and its converse, properties of special right triangles, and right triangle trigonometry)
- G.13 (The student will use formulas for surface area and volume of three-dimensional objects to solve real-world problems.)
- G.14d (Solve real-world problems about similar geometric objects.)

**NCTM Standards**

- Analyze properties and determine attribute of two- and three- dimensional objects
- Explore relationships (including congruence and similarity) among classes of two- and three- dimensional geometric objects, make and test conjectures about them, and solve problems involving them
- Draw and construct representations of two- and three-dimensional geometric objects using a variety of tools.
- Use geometric models to gain insights into, and answer questions in, other areas of mathematics
- Understand and use formulas for the area, surface area, and volume of geometric figures, including cones, spheres, and cylinders.
- Apply and adapt a variety of appropriate strategies to solve problems
- Communicate mathematical thinking coherently and clearly to peers, teachers, and others

**Materials/Resources**
Students will need: Classroom set of Geometry SOL formula sheets, Classroom set of graphing calculators, Classroom set of worksheets

**Assumption of Prior Knowledge**

Students should be familiar with: Volume formulas of various shapes, Pythagorean Theorem, Corresponding parts of similar triangles

## Introduction: Setting Up the Mathematical Task

Teachers should explain that students will be calculating the volume of three different objects. The students will not be given all of the necessary information to immediately plug in into the formulas. Students will need to possibly use other formulas (circumference, Pythagorean Theorem) or similar triangles to find the missing lengths.

Students at lower van Hiele levels would benefit from working with the actual objects (pail, soup can, pyramid). If this is impractical, teachers can create paper models or students can create their own paper models of these objects.

Students will begin with Student Exploration #1. For this portion, students will be finding the volume of three objects (cylinder, pyramid, and section of a cone). Finding the volume of the cylinder and pyramid should not need much teacher instruction. The volume of the frustum for question #2 may need more leading and question asking in order for lower level students to find the volume of the sand pail.

In Student Exploration #2, students will be working with the volumes they found in Student Exploration #1. They will use these volumes to answer questions about which solid has the largest volume and to determine the amount of time for the pail to empty given the rate the pail is losing sand.

In Student Exploration #3, students will be using the given image of a cone to derive the formula for the volume of the frustum. You may wish to use Exploration #3 as enrichment for higher level students or those who finish early. Students will then use their formula to confirm the volume of the pail that they found in Exploration #1.

## Student Exploration

**Small Group Work (Exploration #1)**

Students will work in groups of two or three.

**Student/Teacher Actions (Exploration #1)**

- Students should begin by working in their small groups on questions #1 and #2 of the activity. They should be completing diagrams and calculate volume of the soup can and pyramid for #1.
- To facilitate learning, teachers should be walking around the room assisting any students who may need help with the diagrams or volume.

- There may be many questions about the volume of the pail in #2 since the volume of a frustum is not a formula provided on their Virginia SOL formula sheets. It may help to draw the picture of a cone on the board and show them that the volume of the pail is simply the volume of the larger cone minus the volume of the smaller cone.
- In order to find the volume of the pail, the students may need help labeling the above cone. Students may use similar triangles (or other methods) to solve for the missing lengths. Students will also need the Pythagorean Theorem to solve for the height of the pail/cone which is needed for the volume formula.
- For groups who are finished with this portion of the activity, they can be assigned to other groups to assist them.
- The teacher will need to check the accuracy of the volumes found. Student Exploration #2 will use these volumes to answer questions.

**Monitoring Student Responses (Exploration #1)**

- The teacher will assist students by answering questions and providing suggestions along the way.
- Students should communicate their thinking verbally with their group and in written form by showing their work on their handout or a separate piece of paper.

**Small Group Work (Exploration #2)**

Students will continue working in the same small groups.

**Student/Teacher Actions (Exploration #2)**

- Now that the students have found the volume of the pail, pyramid, and soup can, they should move on to #3 and #4 of the activity.
- If the students need help with the rate problem in #4, it may help to give an example. Suppose a 10 fl oz cup was looking water at a rate of 1 fl oz per second. How many seconds would it take for the cup to empty? The students may be able to relate the process to the pail and sand.
- To facilitate learning, teachers should be walking around the room assisting any students who may need help with the diagrams or volume.

**Monitoring Student Responses (Exploration #2)**

- The teacher will assist students by answering questions and providing suggestions along the way.
- Students should communicate their thinking verbally with their group and in written form by showing their work on their handout or a separate piece of paper.

**Small Group Work (Exploration #3)**

Students will continue working in the same small groups.

**Student/Teacher Actions (Exploration #3)**

- After students have finished Explorations #1 and #2 and confirmed their solutions with the teacher, they should move on to questions #5 and #6.
- For question #5, students will be deriving the formula for the volume of a frustum in terms of *R*, *r*, and *h*. Students may want to leave their formula in terms of *R*, *r*, *h*, and *H*; however, this is not the goal. You may need to explain that you do not always know the height of the original cone that a frustum was constructed from. Therefore, students need to use the proportion given to solve for *H* and substitute this in the formula and simplify.
- Different groups may come up with various forms of the formula for the frustum. All formulas should be algebraically equivalent though.
- Students will verify their formulas using the measurements of their pail from Exploration #1.
- As an extension, the teacher may wish to put some of the formulas on the board and have students show that the formulas are indeed algebraically equivalent although some of the formulas look different.
- To facilitate learning, teachers should be walking around the room assisting any students who may need help with the algebraic steps.

**Monitoring Student Responses (Exploration #3)**

- The teacher will assist students by answering questions and providing suggestions along the way.
- Students should communicate their thinking verbally with their group and in written form by showing their work on their handout or a separate piece of paper.

**Small Group Work (Exploration #4)**

Students will continue working in the same small groups.

**Student/Teacher Actions (Exploration #4)**

- For the final exploration, students will be creating a sketch of a sand castle that incorporates the 3D objects (pail, pyramid, and soup can).
- Students can draw their sketch from the view of their choice (front view, top view, etc).
- The teacher may want to specify a minimum number of buildings the sand castle may have.
- Next, the students will be calculating the total volume of the sand castle. Students can use their calculations from #1 and #2. Students should specify how many of each object they used and show their calculations.
- To facilitate learning, teachers should be walking around the room assisting any students who may need help with the algebraic steps.

**Monitoring Student Responses (Exploration #4)**

- The teacher will assist students by answering questions and providing suggestions along the way.
- Students should communicate their thinking verbally with their group and in written form by showing their work on their handout or a separate piece of paper.

## Assessment List and Benchmarks

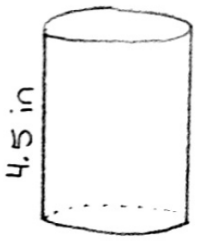
Attached are student worksheets, rubrics, and benchmarks for the task.

# A Day at the Beach!

You are at the beach with your friends. You have brought some supplies to make sand castles. These supplies include a pail that has a base with a circumference of 6π inches, is 10 inches tall, has a slant height of 10.44 inches, and has an opening on top that is twice the diameter of the base. You also have a plastic pyramid mold that has a square base with an edge that measures 4 inches and a slant height of 5.385 inches, and an empty soup can with a base diameter of 3.25 inches and is 4.5 inches tall.

***For each question, include correct units of measurement and round your answer to nearest thousandth or in terms of π.***

1) Draw and label diagrams that represent the soup can and pyramid. Calculate how much sand you can fit into each object.



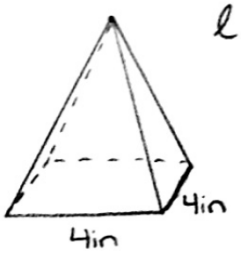

$D = 3.25\text{ in}$

$r = 1.625\text{ in}$

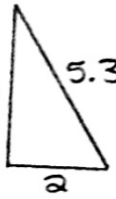
$V = \pi r^2 h$

$= \pi(1.625)^2(4.5)$

$\approx \boxed{11.88\pi\text{ in}^3 \text{ or } 37.33\text{ in}^3}$




$\ell = 5.385\text{in}$




$h^2 + 2^2 = 5.385^2$

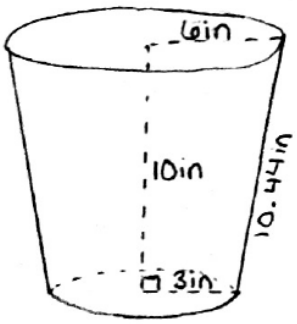
$h = 5\text{in}$

$V = \frac{1}{3}Bh$

$= \frac{1}{3}(16)(5)$

$= \boxed{26.667\text{ in}^3}$

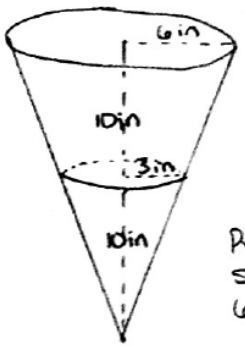
2) Draw and label a diagram that could represent the pail. Calculate how much sand will fit in the pail. (Hint: Similar triangles may help.)




$C = 6\pi$
$d = 6\text{in}$ Base
$r = 3\text{in}$

diam of top = 12 in

$r = 6\text{in}$




Pyth. Thm to solve for y:

$6^2 + b^2 = 20.88^2$

$b = 20$

So $y = 10$

$V_{LARGE} = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi(6)^2(20)$

$= 240\pi\text{ in}^3$

$V_{SMALL} = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi(3)^2(10)$

$= 30\pi$

Pail Volume =

$240\pi - 30\pi$

$= \boxed{210\pi\text{ in}^3 \text{ or } 659.734\text{ in}^3}$

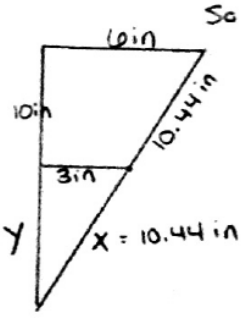
$\frac{6}{10.44 + x} = \frac{3}{x}$

$3(10.44 + x) = 6x$

$31.32 + 3x = 6x$

$31.32 = 3x$

$x = 10.44$



3) You and your friend are going to have a race to see who can fill the pail the fastest. Which object would you choose, the soup can or the plastic pyramid? Justify your answer and determine how many times you would need to empty your object to fill the pail.

Volume Pail ÷ Volume Can
= 659.734 ÷ 37.33
= 17.673

Volume Pail ÷ Volume Pyramid
= 659.734 ÷ 26.6667
= 24.740

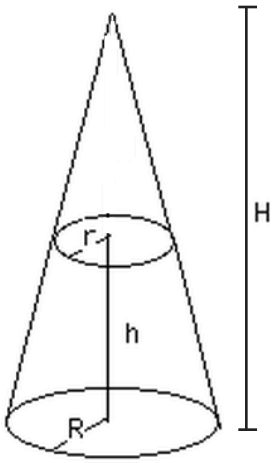
I would choose the soup can because the volume of the can is larger than the volume of the pyramid. Therefore I would need fewer scoops to fill the pail. I would need 17.673 or 18 scoops to fill the pail.

4) If the pail has a hole in the bottom and is losing sand at a rate of 5.3 $in^3$ per second, how long would it take a full pail to completely empty?

Volume Pail ÷ Rate = Time to empty
659.734 ÷ 5.3 = 124.478

It will take 124.478 seconds for the pail to empty.

5) Use the image to the right to derive the formula for the volume of a frustum. Simplify your formula.




By similar triangles you know that $\frac{H}{h} = \frac{H-h}{r}$. Use this fact to write your formula in terms of *R*, *r*, and *h*.

Big Cone - Small Cone

$\frac{1}{3}\pi R^2 H - \frac{1}{3}\pi r^2 (H-h)$

$\frac{1}{3}\pi R^2 H - \frac{1}{3}\pi r^2 H + \frac{1}{3}\pi r^2 h$

$\frac{1}{3}\pi (R^2 H - r^2 H + r^2 h)$

$\frac{1}{3}\pi \left(R^2\left(\frac{RH}{R-r}\right) - r^2\left(\frac{Rh}{R-r}\right) + r^2 h\right)$ ← substitute

$\frac{1}{3}\pi \left(\frac{R^3 h}{R-r} - \frac{Rr^2 h}{R-r} + r^2 h\right)$

$\frac{1}{3}\pi \left(\frac{R^3 h - Rr^2 h}{R-r} + r^2 h\right)$

$\frac{1}{3}\pi \left(\frac{Rh(R^2-r^2)}{R-r} + r^2 h\right)$

$\frac{1}{3}\pi \left(\frac{Rh(R+r)(R-r)}{R-r} + r^2 h\right)$

$\frac{1}{3}\pi (R^2 h + Rrh + r^2 h$

$\frac{1}{3}h\pi (R^2 + Rr + r^2)$

$\frac{H}{R} = \frac{H-h}{r}$

$rH = R(H-h)$

$rH = RH - Rh$

$Rh = RH - rH$

$Rh = H(R-r)$

$\frac{Rh}{R-r} = H$

6) Verify your formula from question 5 by finding the volume of the sand pail.

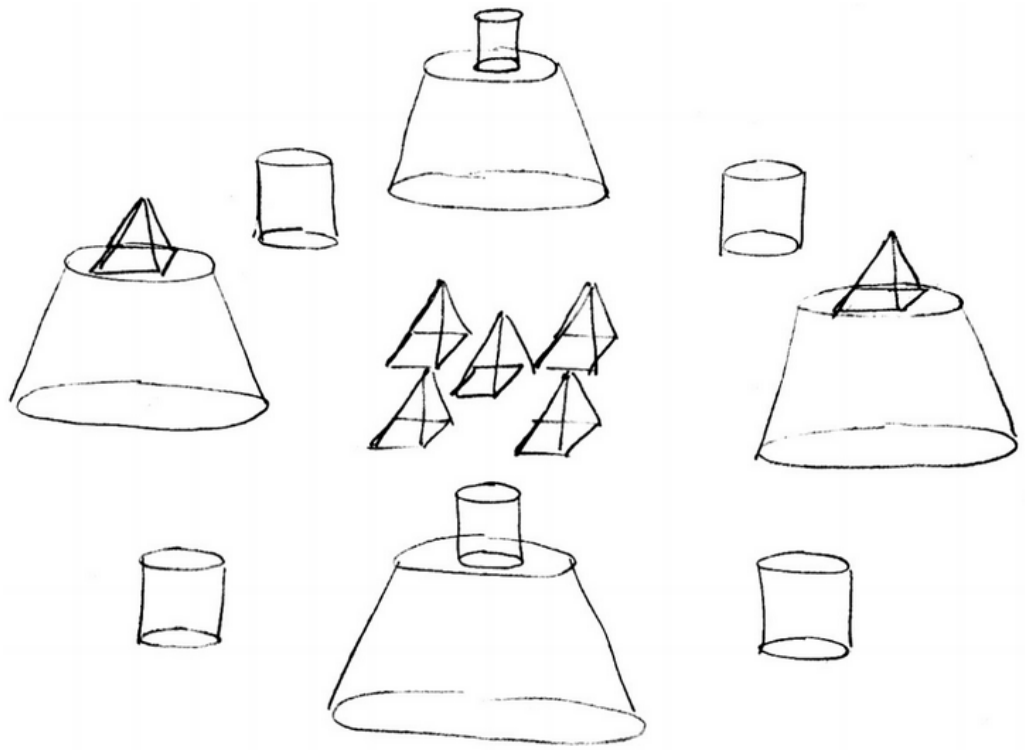
$\frac{1}{3}h\pi (R^2 + Rr + r^2)$ $\quad r = 3 \quad R = 6 \quad h = 10$

$\frac{1}{3}(10)\pi (6^2 + (6)(3) + 3^2)$

$\frac{10}{3}\pi (63)$

$210\pi$ OR $\boxed{659.734\ in^3}$

7) You and your friends are going to build a sand castle that incorporates the pail, pyramid, and soup can. Create a sketch of a sand castle that incorporates these 3-D objects.



8) Calculate the total volume required of sand to build the entire sand castle.

6 Cones: $37.33 \times 6 = 223.98\ in^3$

7 Pyramids: $26.667 \times 7 = 1866.669\ in^3$

4 Pails: $659.734 \times 4 = 2638.936\ in^3$

Total = $4729.585\ in^3$

The sand castle will need $4729.585\ in^3$ of sand.

Students will be graded with the following rubric:

| | 3 | 2 | 1 | 0 |
|---|---|---|---|---|
| **#1: Diagrams for soup can and pyramid drawn and measurements labeled** | Complete and accurate. | Partially complete and mostly accurate. | Attempted completion, but incorrect. | Not complete. |
| **#1: Volume for soup can and pyramid** | All calculations shown, complete, and accurate. | Partial calculations are shown, complete, and accurate. | Attempted to complete calculations but incorrect. | Not complete. |
| **#2: Diagram for sand pail and measurements** | Complete and accurate. | Partially complete and mostly accurate. | Attempted completion, but incorrect. | Not complete. |
| **#2: Volume of sand pail** | All calculations shown, complete, and accurate. | Partial calculations are shown, complete, and accurate. | Attempted to complete calculations but incorrect. | Not complete. |
| **#3: Answer and justification for chosen object** | Used precise mathematical language to clearly communicate thinking. | Partially communicated thinking and explanation. | Used minimal communication and explanation. | Not complete. |
| **#4: Calculation of time for pail to completely empty** | All calculations shown, complete, and accurate. | Partial calculations are shown, complete, and accurate. | Attempted to complete calculations but incorrect. | Not complete. |
| **#5: Derive formula for volume of frustum.** | Complete and accurate. | Partially complete and mostly accurate. | Attempted completion, but incorrect. | Not complete. |
| **#6: Confirm formula by finding volume of pail.** | All calculations shown, complete, and accurate. | Partial calculations are shown, complete, and accurate. | Attempted to complete calculations but incorrect. | Not complete. |
| **#7: Sketch of sand castle incorporating 3D objects.** | Diagram complete and accurate. | Partially complete and mostly accurate. | Attempted completion, but incorrect. | Not complete. |
| **#8: Calculation of total volume of sand castle.** | All calculations shown, complete, and accurate. | Partial calculations are shown, complete, and accurate. | Attempted to complete calculations but incorrect. | Not complete. |

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***For each question, include correct units of measurement and round your answer to nearest thousandth or in terms of π.***

9) Draw and label diagrams that represent the soup can and pyramid. Calculate how much sand you can fit into each object.

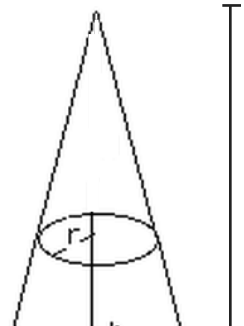
10) Draw and label a diagram that could represent the pail. Calculate how much sand will fit in the pail. (Hint: Similar triangles may help.)

11) You and your friend are going to have a race to see who can fill the pail the fastest. Which object would you choose, the soup can or the plastic pyramid? Justify your

answer and determine how many times you would need to empty your object to fill the pail.

12) If the pail has a hole in the bottom and is losing sand at a rate of 5.3 $in^3$ per second, how long would it take a full pail to completely empty?

13) Use the image to the right to derive the formula for the volume of a frustum. Simplify your formula.



By similar triangles you know that $\frac{H}{h} = \frac{H-h}{r}$. Use this fact to write your formula in terms of *R*, *r*, and *h*.

H

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14) Verify your formula from question 5 by finding the volume of the sand pail.

15) You and your friends are going to build a sand castle that incorporates the pail, pyramid, and soup can. Create a sketch of a sand castle that incorporates these 3-D objects.

16) Calculate the total volume required of sand to build the entire sand castle.